## GCE Examinations

# Further Pure Mathematics Module FP2

Advanced Subsidiary / Advanced Level

## Paper B

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.



Written by Rosemary Smith & Shaun Armstrong © Solomon Press

These sheets may be copied for use solely by the purchaser's institute.

1. Given that

$$y \arccos x - \frac{x}{\pi} e^{2x} - 1 = 0,$$

find the value of  $\frac{dy}{dx}$  at the point where x = 0, giving your answer in terms of  $\pi$ . (7 marks)

 $f(x) = 5\cosh x + 3\sinh x.$ 

The minimum value of f(x) occurs at the point  $(p \ln q, r)$  where p, q and r are integers.

Find the values of p, q and r.

(8 marks)

3. The line y = mx + c is a tangent to the rectangular hyperbola with equation xy = -9.

- (a) Show that  $c = \pm 6\sqrt{m}$ . (4 marks)
- (b) Hence, or otherwise, find the equations of the tangents from the point (4, -2) to the rectangular hyperbola xy = -9.

(5 marks)

4. The curve *C* is defined by

$$y^2 = x, \qquad x \ge 0, \ y \ge 0.$$

The region between *C*, the *x*-axis and the line x = 1 is rotated through  $2\pi$  about the *x*-axis.

Show that the area of the surface generated is

$$\frac{\pi}{6}(5\sqrt{5}-1).$$
 (11 marks)

- 5. (a) Using the definition of  $\cosh x$  in terms of exponential functions, express sech x in terms of  $e^x$  and  $e^{-x}$ .
  - (b) Sketch the graph of  $y = \operatorname{sech} x$ .(1 mark)(c) Show that  $\int \operatorname{sech} x \, dx = 2 \arctan e^x + c$ .(4 marks)

The curve *C* has equation  $y = \operatorname{sech} x$ . The region between *C*, the *x*-axis and the lines x = -a and x = a, where *a* is a positive constant, is rotated through  $2\pi$  about the *x*-axis.

- (d) Find the volume of revolution of the solid generated. (4 marks)
- (e) Find the limit of the volume of revolution as  $a \to \infty$ . (1 mark)

$$I_n = \int_0^{\sqrt{2}} (2 - x^2)^n \, \mathrm{d}x \, , \quad n \ge 0$$

(a) Show that

6.

$$I_n = \frac{4n}{2n+1} I_{n-1}, \quad n \ge 1.$$
 (9 marks)

(b) Hence evaluate  $I_3$ , leaving your answer in surd form. (4 marks)

### 7. The curve *C* has intrinsic equation

 $s = \ln(\tan \frac{1}{2}\psi), \qquad 0 < \psi \le \frac{\pi}{2}.$ 

(a) Show that radius of curvature of C is given by  $\rho = \csc \psi$ . (4 marks)

Given that  $y = \psi = \frac{\pi}{2}$  when x = 0,

- (b) show that  $y = \psi$ , (4 marks)
- (c) use integration to show that a Cartesian equation of C is  $x = \ln(\sin y)$ . (7 marks)

### END